PMM U.S.S.R.,Vol.48,No.1,pp.112-114,1984
Printed in Great Britain

0021-8928/84 $10.00+0.00

SLIP LINES AT THE VERTEX OF A WEDGE-LIKE CUT*

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The problem of the initial development of plastic deformations near a tip of a wedge-like cut in a homogeneous isotropic body is studied under conditions of plane deformation. It is assumed that the plastic deformations concentrate along narrow rectilinear slippage strips emerging from the tip of the cut. The integral Mellin transform is used to reduce the problem to the functional Wiener-Hopf equation, and its closed solution is given. An equation for determining the length of the slip line is derived.

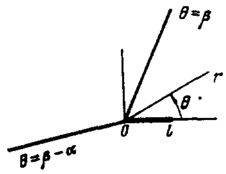

Fig.1

Consider a homogeneous isotropic body with a wedge-like cut. The material of the body is assumed to be perfectly elastoplastic. When the external loads are sufficiently small, the characteristic linear dimension of the plastic zone near the tip of the cut of angle α will be small compared with the characteristic dimension of the body and the cut. We will assume that the plastic deformations are concentrated along the narrow rectilinear slip lines emerging from the tip of the cut. Applying the "microscope principle" /1/ we arrive at the singular problem of the theory of elasticity of class N, concerning the equilibrium of an elastic wedge with the stress-free boundaries and of angle greater than π, with a rectilinear slip line emerging from its tip (Fig.1).

We will write the boundary conditions for this problem as follows:

$$\theta = \beta,\ \theta = \beta - \alpha,\ \sigma_\theta = \tau_{r\theta} = 0;\ \alpha \in (\pi, 2\pi),\ \beta \in (0, \alpha) \tag{1}$$

$$\theta = 0,\ [\sigma_\theta] = [\tau_{r\theta}] = 0,\ [u_\theta] = 0$$

$$\theta = 0,\ r < l,\ \tau_{r\theta} = \tau_s;\ \theta = 0,\ r > l,\ [u_r] = 0 \tag{2}$$

Here $\sigma_\theta, \tau_{r\theta}, \sigma_r$ are the stresses, u_θ, u_r are displacements, $[a]$ is the jump in the value of a, and τ_s is the limiting shear strength. At infinity the solution of the problem behaves like a solution of the canonical singular problem of the theory of elasticity for a wedge $\beta - \alpha < \theta < \beta,\ 0 < r < \infty$ with stress-free edges /1/. In particular we have

$$\theta = 0,\quad r \to \infty,\quad \tau_{r\theta} = C_1 g_1 r^{\lambda_1 - 1} + C_2 g_2 r^{\lambda_2 - 1} + o(1/r)$$

$$g_1 = \frac{1}{2}(2\pi)^{\lambda_1 - 1}\left[(\lambda_1 - 1)\sin(\lambda_1 - 1)\frac{\alpha - 2\beta}{2} - \frac{(\lambda_1 - 1)\sin(\lambda_1 - 1)\alpha/2}{\sin(\lambda_1 + 1)\alpha/2}\sin(\lambda_1 + 1)\frac{\alpha - 2\beta}{2}\right]$$

$$g_2 = \frac{1}{2}(2\pi)^{\lambda_2 - 1}\left[-(\lambda_2 - 1)\cos(\lambda_2 - 1)\frac{\alpha - 2\beta}{2} + \frac{(\lambda_2 + 1)\sin(\lambda_2 - 1)\alpha/2}{\sin(\lambda_2 + 1)\alpha/2}\cos(\lambda_2 + 1)\frac{\alpha - 2\beta}{2}\right]$$

(when $\pi < \alpha \leqslant \alpha_* < 3\pi/2$, the term corresponding to C_2 vanishes). Here $\lambda_j(\alpha) \in (1/2, 1)$ is a unique root of the equation

$$\sin p\alpha - (-1)^j p \sin\alpha = 0\ \ (j = 1, 2;\ \alpha_j < \alpha < 2\pi,\ \alpha_1 = \pi,\ \alpha_2 = \alpha_*)$$

in the strip $0 < \operatorname{Re} p < 1$; α_* is a unique root of the equation $\alpha\cos\alpha - \sin\alpha = 0$ in the interval $\pi < \alpha < 2\pi\ (\alpha_* \approx 257^\circ)$ and C_j are arbitrary real constants.

The constants C_1 and C_2 are assumed given by the condition. The constants characterize the external field intensity and are found from the solution of the outer problem. The constant C_j has dimension of force divided by length to the power $\lambda_j(\alpha) + 1$. The solution of this problem represents the sum of solutions of the following two problems. The first (Problem A) differs from it in the fact that in place of the first condition of (2) we have

$$\theta = 0,\quad r < l,\quad \tau_{r\theta} = \tau_s - C_1 g_1 r^{\lambda_1 - 1} - C_2 g_2 r^{\lambda_2 - 1} \tag{3}$$

and the stresses decay at infinity as $o(1/r)$. The second problem is a canonical singular problem for a wedge with stress-free edges. Since the solution of the second problem is known, it remains to construct the solution for Problem A.

Applying the integral Mellin transform (p is a complex parameter)

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$$m^*(p) = \int_0^\infty m(r)\, r^p\, dr$$

to the equation of equilibrium, condition of compatibility of deformation, Hooke's Law and conditions (1), and taking into account the second condition of (2) and condition (3), we arrive at the functional Wiener-Hopf equation for Problem A

$$\frac{\tau_s}{p+1} + \frac{\tau_1}{p+\lambda_1} + \frac{\tau_2}{p+\lambda_2} + \Phi^+(p) = -\operatorname{tg} p\pi G(p)\, \Phi^-(p), \qquad \tau_j = -C_j g_j l^{\lambda_j - 1}, \quad \Phi^+(p) = \int_1^\infty \tau_{r\theta}(\rho l, 0)\, \rho^p\, d\rho \tag{4}$$

$$\Phi^-(p) = \frac{E}{4(1-\nu^2)} \int_0^1 \left[\frac{\partial u_r}{\partial r}\right]\Bigg|_{\substack{\theta=0\\ r=\rho l}} \rho^p\, d\rho$$

$$G(p) = \frac{4\delta_1\delta_2(\delta_1\Delta_2^+ + \delta_2\Delta_1^+)\cos p\pi}{[4p^2(p^2-1)d^2 + (\delta_1\Delta_2^+ + \delta_2\Delta_1^+)(\delta_1\Delta_2^- + \delta_2\Delta_1^-)]\sin p\pi}$$

$$\delta_j = \sin^2 p\theta_j - p^2 \sin^2\theta_j, \ \Delta_j^\pm = \sin 2p\theta_j \pm p \sin 2\theta_j$$

$$d = \sin^2 p\theta_1 \sin^2\theta_2 - \sin^2 p\theta_2 \sin^2\theta_1, \ \theta_1 = \beta, \ \theta_2 = \alpha - \beta$$

(E is Young's modulus and ν is Poisson's ratio). The solution of (4) is constructed in the same manner as those of the functional Wiener-Hopf equations for the problems discussed in /2, 3/. We have ($\Gamma(z)$ is the gamma function)

$$\Phi^-(p) = K^-(p)\, G^-(p) \left[\frac{\tau_s K^+(-1)}{(p+1)G^+(-1)} + \sum_{j=1}^{2} \frac{\tau_j K^+(-\lambda_j)}{\lambda_j (p+\lambda_j) G^+(-\lambda_j)}\right] \quad (\operatorname{Re} p > 0) \tag{5}$$

$$\Phi^+(p) = -\frac{pG^+(p)}{K^+(p)} \left\{\frac{\tau_s}{p+1}\left[\frac{K^+(p)}{pG^+(p)} + \frac{K^+(-1)}{G^+(-1)}\right] + \sum_{j=1}^{2} \frac{\tau_j}{p+\lambda_j}\left[\frac{K^+(p)}{pG^+(p)} + \frac{K^+(-\lambda_j)}{\lambda_j G^+(-\lambda_j)}\right]\right\} \quad (\operatorname{Re} p < 0)$$

$$\exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{\ln G(t)}{t-p}\, dt\right] = \begin{cases} G^+(p), & \operatorname{Re} p < 0 \\ G^-(p), & \operatorname{Re} p > 0 \end{cases}$$

$$K^\pm(p) = \frac{\Gamma(1 \mp p)}{\Gamma(^1/_2 \mp p)}$$

We will find the stress intensity coefficient k_{II} at the head of the slip line From the first formula of (5) we obtain

$$\Phi^-(p) \sim \frac{Q}{\sqrt{p}} \quad (p \to \infty) \tag{6}$$

$$Q = \frac{\tau_s K^+(-1)}{G^+(-1)} + \sum_{j=1}^{2} \frac{\tau_j K^+(-\lambda_j)}{\lambda_j G^+(-\lambda_j)}$$

and we have the asymptotics

$$\Phi^-(p) \sim -\frac{k_{\text{II}}}{\sqrt{2pl}} \quad (p \to \infty) \tag{7}$$

We will use (6) and (7) to obtain the coefficient k_{II} Equating it to the viscous slippage k_{IIc}, which is a given material constant, we obtain the following equation for determining the length l of the slip line

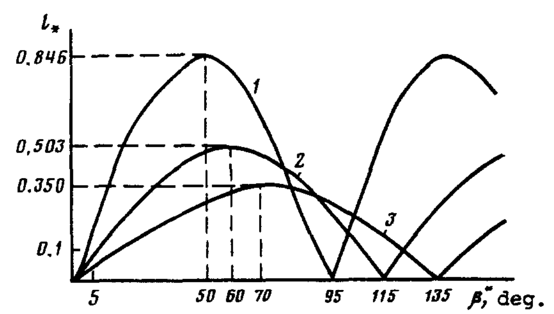

Fig.2

$$\sum_{j=1}^{2} L_j(\alpha,\beta)\, C_j l^{\lambda_j - 1/2} - \tau_s \sqrt{l} = \frac{\sqrt{\pi}}{2\sqrt{2}} k_{\text{IIc}} G^+(-1) \tag{8}$$

$$L_j(\alpha,\beta) = \frac{\sqrt{\pi}\, g_j \Gamma(1+\lambda_j)\, G^+(-1)}{2\lambda_j \Gamma(^1/_2 + \lambda_j)\, G^+(-\lambda_j)}$$

(when $\pi < \alpha \leqslant \alpha_*$, the term corresponding to C_2 vanishes).

Let us assume that the head of the slip line is free, i.e. $k_{\text{IIc}} = 0$. Let $C_2 = 0$. From (8) we find, for $k_{\text{IIc}} = 0$,

$$\tau_s l^{1-\lambda_1}/C_1 = L_1(\alpha, \beta)$$

Fig.2 shows the dependence of $l_* = |\tau_s l^{1-\lambda_1}/C_1|$ on the angle β at $\alpha = 190, \ 230, 270°$ (curves *1*, *2* and *3* are symmetrical about the straight lines $\beta = \alpha/2$ respectively). Following /3/ we assume that in a body homogeneous and isotropic with respect to its strength, the slip lines near the cut tip develop in the direction of the largest value of l_*. Analyzing the relationship given we conclude that in the case in question ($C_2 = 0$) the slip lines will develop in two directions that are symmetrical about the bisector plane of the wedge. The values of the angle $\bar\beta$ of inclination of the slip line to the wedge edge, are given below for several values of the wedge angle α

α, deg.= 190 210 230 250 270 290 310 330 350
β, deg.= 50 53 60 66 70 80 87 94 105

Note. It was pointed out by V.K. Vostrov that for $G^+(-1/_2)$ of /3/ the factor $\sqrt{2}(\sqrt{3}\sin\alpha\cos\alpha/2)^{-1}$ should be multiplied by the inverse expression $\sqrt{3/_2}\sin\alpha\cos\alpha/2$. The numerical factors in (4.9) and (4.12) will now become 0.058 and 0.28 respectively (compared with the previous 0.046 and 0.22).

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Translated by L.K.

PMM U.S.S.R.,Vol.48,No.1,pp.114-117,1984
Printed in Great Britain

0021-8928/84 $10.00+0.00

ON A STAR-LIKE SYSTEM OF PROPAGATING DISLOCATION DISCONTINUITIES*

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An antiplane dynamic problem of a system of dislocation discontinuities propagating from the origin of coordinates and forming a star-like structure is considered. A displacement field is obtained and specific features of seismic radiation in the far zone are studied.

Let $2n$ dislocation discontinuities with uniform angular distribution (Fig.1) begin to propagate at the initial instant $t=0$ from the origin of a Cartesian system of coordinates Oxy, with constant velocity, in an isotropic elastic medium. We define the discontinuity kinematically, i.e. we specify at each point of the plane of discontinuity the magnitude and direction of the displacement jump vector at the discontinuity, depending on the coordinates and time. As was shown in /1–6/, the kinematic description of the discontinuities shows in many cases a number of preferences as compared with the dynamic method whereby the forces are defined at the discontinuity. An analogous problem for the cracks using the dynamic method of describing the discontinuities was studied in /7/.

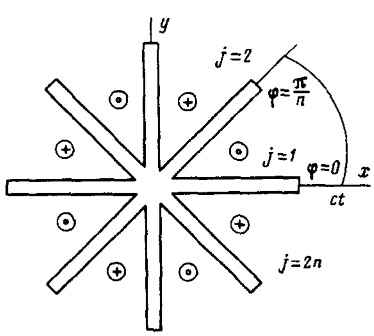

Fig.1

We shall assume that every single dislocation discontinuity is described by a symmetric (about the plane of discontinuity) homogeneous function of zero dimension $f(\rho/t)$. We denote by σ_{xz} and σ_{yz} the stress tensor components and by w the unique non-zero displacement vector component satisfying the wave equation

$$\frac{\partial^2 w}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2 w}{\partial\varphi^2}=\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} \tag{1}$$

$$\left(\rho=\sqrt{x^2+y^2},\ \varphi=\operatorname{arctg}\frac{y}{x}\right)$$

where ρ, φ are polar coordinates and c is the velocity of transverse waves. The boundary conditions are

$$\left.\begin{matrix}[w]=f(\rho/t), & \rho\leqslant vt\\ [w]=0. & \rho>vt\end{matrix}\right\}\varphi=0,\frac{\pi}{n};\quad n=1,2,3\ldots \tag{2}$$

Thus we must find a solution of problem (1), (2) belonging to the class of selfsimilar problems with the selfsimilarity index (0,0). We use the Smirnov-Sobolev method /3/ of the functionally invariant solutions, and the general approach employed in solving such problems /9/, enabling us to reduce the selfsimilar problems of the dynamic theory of elasticity to the boudary value problems of the theory of analytic functions.

*Prikl.Matem.Mekhan.,48,1,163-166,1984